Series 5: solutions

Exercise 1

We consider a linear array of 5 equidistant antennas separated by a distance d = λ/2. The array feeds are symmetrical (I0 = I4 and I1 = I3) and normalized with respect to the current of the central antenna (I2 = 1). We consider that the array is placed on the z-axis (figure).

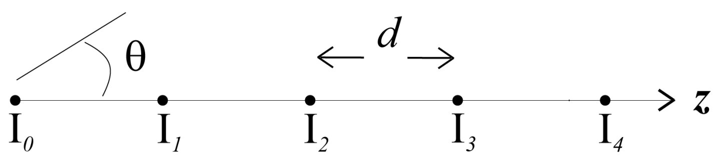

Figure 1. Réseau linéaire équidistant.

We want to obtain the following characteristics for the normalized array factor (NAF):

a) NAF 0dB for θ =90 b) NAF=-∞ dB for θ =60 c) NAF=-10dB for θ =0 (angles in degrees).

Find the values of currents I0 and I1 .

	Amplitude	Phase
I_0		
I_1		

Note: It can be intuitively assumed that the maximum radiation is in the "broadside" direction (condition a)) and verified a posteriori by plotting the diagram obtained in Matlab once the coefficients have been found using the other conditions.

Solution:

According to the initial condition, we have

$$\boxed{I_2 = 1}$$

$$d = \frac{\lambda}{2}, \qquad kd = \pi.$$

The AF array factor and the NAF normalized array factor for a linear equidistant network are:

$$AF(\theta) = \sum_{n=0}^{N-1} I_n e^{jnkd\cos\theta}, \qquad I_n = A_n e^{j\alpha_n}$$

$$NAF(\theta) = \left|\frac{AF(\theta)}{\max[AF(\theta)]}\right| = \left|\frac{I_0 + I_1 e^{j\pi\cos\theta} + I_2 e^{j2\pi\cos\theta} + I_1 e^{j3\pi\cos\theta} + I_0 e^{j4\pi\cos\theta}}{\max[AF(\theta)]}\right|$$

After observing the desired characteristics for the NAF, we see that the maximum radiation is in the "broadside" direction, i.e.:

$$NAF(90°) = 0\ \text{dB} = 1 \Rightarrow \alpha = 0$$

$$|I_2 + 2I_1 + 2I_0| = \max[AF(\theta)]$$

Thus, the NAF can be written as :

$$\left|e^{j2\pi cos\theta}\right| = 1 \ \Rightarrow NAF(\theta) = \left|\frac{I_2 + 2I_1\cos(\pi\cos\theta) + 2I_0\cos(2\pi\cos\theta)}{I_2 + 2I_1 + 2I_0}\right|$$

To obtain minimum radiation, it is necessary that:

$$NAF(60°) = -\infty\ \text{dB} = 0$$

$$I_2 - 2I_0 = 0$$

$$\boxed{I_0 = \frac{1}{2}}$$

So, the condition that NAF=-10 dB for θ=0° is :

$$NAF_{dB} = 20\log_{10} NAF$$

$$NAF(0°) = -10\text{ dB} = 10^{-\frac{1}{2}}$$

$$\frac{|I_2 - 2I_1 + 2I_0|}{I_2 + 2I_1 + 2I_0} = \frac{1}{\sqrt{10}}$$

$$\boxed{I_1 = \frac{\sqrt{10}-1}{\sqrt{10}+1} = \frac{11-2\sqrt{10}}{9} \approx\ 0.5195}$$

Th eother solution for $\frac{|I_2-2I_1-2I_0|}{I_2+2I_1+2I_0} = \frac{1}{\sqrt{10}}$ is $I_1 \approx 1.95$. Since the excitation currents are normalized with respect to $I_2 = 1$, this solution is not acceptable.

Elément	$\boldsymbol{|I_i|}$	***phase*(*I*$_i$)**
0	0.5	0
1	≈ 0.5195	0
2	1	0
3	≈ 0.5195	0
4	0.5	0

Table 1. Solution pour the analytic computation.

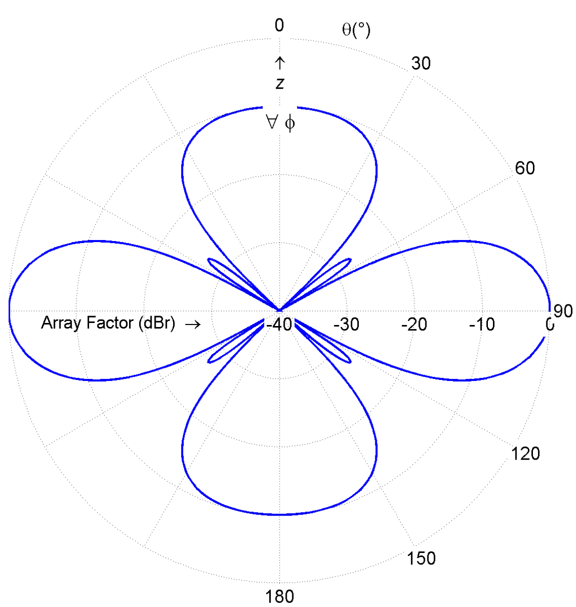

Figure 2: Normalized array factor.

Exercise 2

Consider a linear array of N antennas equidistant and separated by a distance d. The antennas are numbered n=0,1,2...,N-1 and fed with currents $I_n = A^n$. Find the mathematical expression for the array factor (AF). Find, when N=3 and $d = \lambda / 2$, the zero radiation directions the cases A=0.5, 1, and 2.

Solution:

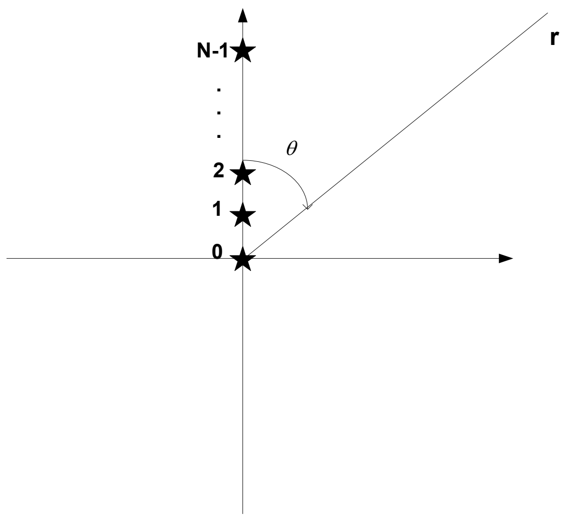

Figure 1 : Array geometry

The array factor AF is given by :

$$\text{AF} = \sum_{n=0}^{N-1} I_n e^{jk\hat{e}_r \cdot \vec{d}_n} .$$

According to the array geometry (Figure 1), we have that :

$$\vec{d}_n = nd\hat{z} \Rightarrow \hat{e}_r \cdot \vec{d}_n = \left(\hat{e}_r \cdot \hat{z}\right) nd = nd\cos\theta .$$

Thus,

$$\mathrm{AF} = \sum_{n=0}^{N-1} A^n e^{jknd\cos\theta} = \sum_{n=0}^{N-1} \left(Ae^{jkd\cos\theta} \right)^n .$$

This geometric series yields :

$$\mathrm{AF} = \frac{1 - A^N e^{jkNd\cos\theta}}{1 - Ae^{jkd\cos\theta}} .$$

In teh case where $d=\lambda/2$, AF is give by :

$$d = \lambda/2 \Rightarrow \mathrm{AF} = \frac{1 - A^N e^{jN\pi\cos\theta}}{1 - Ae^{j\pi\cos\theta}} .$$

The zero radiation directions are found as:

$$AF = 0 \Longrightarrow \frac{1 - A^N e^{jN\pi\, cos\theta}}{1 - Ae^{j\pi\, cos\theta}} = 0 \Longrightarrow A^N e^{jN\pi\, cos\theta} = 1.$$

Therefore, equality is only possible if A = 1. In this case, we can rewrite AF:

$$\mathrm{AF} = \frac{1 - e^{jN\pi\cos\theta}}{1 - e^{j\pi\cos\theta}} = \frac{e^{j\frac{N}{2}\pi\cos\theta}}{e^{j\frac{1}{2}\pi\cos\theta}} \frac{e^{-j\frac{N}{2}\pi\cos\theta} - e^{j\frac{N}{2}\pi\cos\theta}}{e^{-j\frac{1}{2}\pi\cos\theta} - e^{j\frac{1}{2}\pi\cos\theta}} = \frac{e^{j\frac{N}{2}\pi\cos\theta}}{e^{j\frac{1}{2}\pi\cos\theta}} \frac{\sin\left(\frac{N}{2}\pi\cos\theta\right)}{\sin\left(\frac{1}{2}\pi\cos\theta\right)}$$

When the sine argument is small, AF will look like:

$$AF \simeq \frac{\sin\left(\frac{N}{2}\pi\cos\theta\right)}{\frac{\pi}{2}\cos\theta} .$$

Then, the radiation is zero when:

$$\sin\left(\frac{N}{2}\pi\cos\theta\right) = 0 \Rightarrow \frac{N}{2}\pi\cos\theta = \pm k\pi \qquad \begin{matrix} \mathrm{k} = 1 \\ \mathrm{k} \neq 0 \end{matrix}$$

For k=0, AF reaches its maximum because it transforms into sin0/0.

So, we see that the zero-radiation directions in the case N=3 and d=λ/2 are given by:

$$\sin\left(\frac{3}{2}\pi\cos\theta\right)=0 \Rightarrow \frac{3}{2}\pi\cos\theta=\pm\pi$$

$$\theta=\arccos\left(\pm\frac{2}{3}\right) \Rightarrow \theta=48.19^\circ \text{ ou } \theta=131.81^\circ .$$